**Chapter 13**

**Experimental Design and**

**Analysis of Variance**

**Learning Objectives**

1. Understand the basic principles of an experimental study.

2. Understand the difference between a completely randomized design, a randomized block design, and a factorial experiment.

3. Know the assumptions necessary to use the analysis of variance procedure.

4. Understand the use of the *F* distribution in performing the analysis of variance procedure.

5. Know how to set up an ANOVA table and interpret the entries in the table.

6. Know how to use the analysis of variance procedure to determine if the means of more than two populations are equal for a completely randomized design, a randomized block design, and a factorial experiment.

7. Know how to use the analysis of variance procedure to determine if the means of more than two populations are equal for an observational study.

8. Be able to use output from computer software packages to solve experimental design problems.

9. Know how to use Fisher’s least significant difference (LSD) procedure and Fisher’s LSD with the Bonferroni adjustment to conduct statistical comparisons between pairs of population means.

**Solutions:**

1. a. = (156 + 142 + 134)/3 = 144

= 6(156 – 144) 2 + 6(142 – 144) 2 + 6(134 – 144) 2 = 1,488

b. MSTR = SSTR /(*k* – 1) = 1488/2 = 744

c.  = 164.4  = 131.2  = 110.4

 = 5(164.4) + 5(131.2) +5(110.4) = 2030

d. MSE = SSE /(*n*T – *k*) = 2030/(12 – 3) = 135.3

e.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 1488 | 2 | 744 | 5.50 | .0162 |
| Error | 2030 | 15 | 135.3 |  |  |
| Total | 3518 | 17 |  |  |  |

f. *F* = MSTR /MSE = 744/135.3 = 5.50

Using *F* table (2 degrees of freedom numerator and 15 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 5.50 is .0162.

Because *p*-value  = .05, we reject the hypothesis that the means for the three treatments are equal.

2.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 300 | 4 | 75 | 14.07 | .0000 |
| Error | 160 | 30 | 5.33 |  |  |
| Total | 460 | 34 |  |  |  |

3. a. *H*0: *u*1 = *u*2 = *u*3 = *u*4 = *u*5

*H*a: Not all the population means are equal

b. Using *F* table (4 degrees of freedom numerator and 30 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 14.07 is .0000.

Because *p*-value  = .05, we reject *H*0

4.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 150 | 2 | 75 | 4.80 | .0233 |
| Error | 250 | 16 | 15.63 |  |  |
| Total | 400 | 18 |  |  |  |

Using *F* table (2 degrees of freedom numerator and 16 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 4.80 is .0233.

Because *p*-value  = .05, we reject the null hypothesis that the means of the three treatments are equal.

5.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 1200 | 2 | 600 | 43.99 | .0000 |
| Error | 600 | 44 | 13.64 |  |  |
| Total | 1800 | 46 |  |  |  |

Using *F* table (2 degrees of freedom numerator and 44 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 43.99 is .0000.

Because *p*-value = .05, we reject the hypothesis that the treatment means are equal.

6.

|  |  |  |  |
| --- | --- | --- | --- |
|  | A | B | C |
| Sample Mean | 119 | 107 | 100 |
| Sample Variance | 146.89 | 96.43 | 173.78 |



= 8(119 – 107.93) 2 + 10(107 – 107.93) 2 + 10(100 – 107.93) 2 = 1617.9

MSTR = SSTR /(*k* – 1) = 1617.9 /2 = 809.95

 = 7(146.86) + 9(96.44) + 9(173.78) = 3,460

MSE = SSE /(*n*T – *k*) = 3,460 /(28 – 3) = 138.4

*F* = MSTR /MSE = 809.95 /138.4 = 5.85

Using *F* table (2 degrees of freedom numerator and 25 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 5.85 is .0082.

Because *p*-value = .05, we reject the null hypothesis that the means of the three treatments are equal.

7. a.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 4560 | 2 | 2280 | 9.87 | .0006 |
| Error | 6240 | 27 | 231.11 |  |  |
| Total | 10800 | 29 |  |  |  |

b. Using *F* table (2 degrees of freedom numerator and 27 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 9.87 is .0006.

Because *p*-value = .05, we reject the null hypothesis that the means of the three assembly methods are equal.

8. = (79 + 74 + 66)/3 = 73

= 6(79 – 73) 2 + 6(74 – 73) 2 + 6(66 – 73) 2 = 516

MSTR = SSTR /(*k* – 1) = 516/2 = 258

 = 34  = 20  = 32

 = 5(34) + 5(20) +5(32) = 430

MSE = SSE /(*n*T – *k*) = 430/(18 – 3) = 28.67

*F* = MSTR /MSE = 258/28.67 = 9.00

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 516 | 2 | 258 | 9.00 | .003 |
| Error | 430 | 15 | 28.67 |  |  |
| Total | 946 | 17 |  |  |  |

Using *F* table (2 degrees of freedom numerator and 15 denominator), *p*-value is less than .01

Using Excel or Minitab the *p*-value corresponding to *F* = 9.00 is .003.

Because *p*-value  = .05, we reject the null hypothesis that the means for the three plants are equal. In other words, analysis of variance supports the conclusion that the population mean examination score at the three NCP plants are not equal.

9.

|  |  |  |  |
| --- | --- | --- | --- |
|  | 50° | 60° | 70° |
| Sample Mean | 33 | 29 | 28 |
| Sample Variance | 32 | 17.5 | 9.5 |

 = (33 + 29 + 28)/3 = 30

= 5(33 – 30) 2 + 5(29 – 30) 2 + 5(28 – 30) 2 = 70

MSTR = SSTR /(*k* – 1) = 70 /2 = 35

 = 4(32) + 4(17.5) + 4(9.5) = 236

MSE = SSE /(*n*T – *k*) = 236 /(15 – 3) = 19.67

*F* = MSTR /MSE = 35 /19.67 = 1.78

Using *F* table (2 degrees of freedom numerator and 12 denominator), *p*-value is greater than .10

Using Excel or Minitab the *p*-value corresponding to *F* = 1.78 is .2104.

Because *p*-value > = .05, we cannot reject the null hypothesis that the mean yields for the three temperatures are equal.

10.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Direct Experience | Indirect Experience | Combination |
| Sample Mean | 17.0 | 20.4 | 25.0 |
| Sample Variance | 5.01 | 6.26 | 4.01 |

= (17 + 20.4 + 25)/3 = 20.8

= 7(17 – 20.8) 2 + 7(20.4 – 20.8) 2 + 7(25 – 20.8) 2 = 225.68

MSTR = SSTR /(*k* – 1) = 225.68 /2 = 112.84

 = 6(5.01) + 6(6.26) + 6(4.01) = 91.68

MSE = SSE /(*n*T – *k*) = 91.68 /(21 – 3) = 5.09

*F* = MSTR /MSE = 112.84 /5.09 = 22.17

Using *F* table (2 degrees of freedom numerator and 18 denominator), *p*-value is less than .01

Using Excel or Minitab the *p*-value corresponding to *F* = 22.17 is .0000.

Because *p*-value = .05, we reject the null hypothesis that the means for the three groups are equal.

11.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Paint 1 | Paint 2 | Paint 3 | Paint 4 |
| Sample Mean | 13.3 | 139 | 136 | 144 |
| Sample Variance | 47.5 | .50 | 21 | 54.5 |

= (133 + 139 + 136 + 144)/3 = 138

= 5(133 – 138) 2 + 5(139 – 138) 2 + 5(136 – 138) 2 + 5(144 – 138) 2 = 330

MSTR = SSTR /(*k* – 1) = 330 /3 = 110

 = 4(47.5) + 4(50) + 4(21) + 4(54.5) = 692

MSE = SSE /(*n*T – *k*) = 692 /(20 – 4) = 43.25

*F* = MSTR /MSE = 110 /43.25 = 2.54

Using *F* table (3 degrees of freedom numerator and 16 denominator), *p*-value is between .05 and .10

Using Excel or Minitab the *p*-value corresponding to *F* = 2.54 is .0931.

Because *p*-value > = .05, we cannot reject the null hypothesis that the mean drying times for the four paints are equal.

12.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Italian | Seafood | Steakhouse |
| Sample Mean | 17 | 19 | 24 |
| Sample Variance | 14.857 | 13.714 | 14.000 |

= (17 + 19 + 24)/3 = 20

= 8(17 – 20) 2 + 8(19 – 20) 2 + 8(24 – 20) 2 = 208

MSTR = SSTR /(*k* – 1) = 208/2 = 104

 = 7(14.857) + 7(13.714) + 7(14.000) = 298

MSE = SSE /(*n*T – *k*) = 298 /(24 – 3) = 14.19

*F* = MSTR /MSE = 104 /14.19 = 7.33

Using the *F* table (2 degrees of freedom numerator and 21 denominator), the *p*-value is less than .01.

Using Excel or Minitab the *p*-value corresponding to *F* = 7.33 is .0038.

Because *p*-value = .05, we reject the null hypothesis that the mean meal prices are the same for the three types of restaurants.

13. a.  = (30 + 45 + 36)/3 = 37

= 5(30 – 37)2 + 5(45 – 37)2 + 5(36 – 37)2 = 570

MSTR = SSTR /(*k* – 1) = 570/2 = 285

 = 4(6) + 4(4) + 4(6.5) = 66

MSE = SSE /(*n*T – *k*) = 66/(15 – 3) = 5.5

*F* = MSTR /MSE = 285/5.5 = 51.82

Using *F* table (2 degrees of freedom numerator and 12 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 51.82 is .0000.

Because *p*-value = .05, we reject the null hypothesis that the means of the three populations are equal.

b. 

LSD; significant difference

 LSD; significant difference

 LSD; significant difference

c. 



–15  3.23 = –18.23 to –11.77

14. a.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Sample 1 | Sample 2 | Sample 3 |
| Sample Mean | 51 | 77 | 58 |
| Sample Variance | 96.67 | 97.34 | 81.99 |

 = (51 + 77 + 58)/3 = 62

= 4(51 – 62) 2 +4(77 – 62) 2 + 4(58 – 62) 2 = 1,448

MSTR = SSTR /(*k* – 1) = 1,448/2 = 724

 = 3(96.67) + 3(97.34) + 3(81.99) = 828

MSE = SSE /(*n*T – *k*) = 828/(12 – 3) = 92

*F* = MSTR /MSE = 724/92 = 7.87

Using *F* table (2 degrees of freedom numerator and 9 denominator), *p*-value is between .01 and .025

Actual *p*-value = .0106

Because *p*-value  = .05, we reject the null hypothesis that the means of the three populations are equal.

b. 

LSD; significant difference

LSD; no significant difference

LSD; significant difference

15. a.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Manufacturer 1 | Manufacturer 2 | Manufacturer 3 |
| Sample Mean | 23 | 28 | 21 |
| Sample Variance | 6.67 | 4.67 | 3.33 |

= (23 + 28 + 21)/3 = 24

= 4(23 – 24) 2 + 4(28 – 24) 2 + 4(21 – 24) 2 = 104

MSTR = SSTR /(*k* – 1) = 104/2 = 52

 = 3(6.67) + 3(4.67) + 3(3.33) = 44.01

MSE = SSE /(*n*T – *k*) = 44.01/(12 – 3) = 4.89

*F* = MSTR /MSE = 52/4.89 = 10.63

Using *F* table (2 degrees of freedom numerator and 9 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 10.63 is .0043

Because *p*-value  = .05, we reject the null hypothesis that the mean time needed to mix a batch of material is the same for each manufacturer.

b. 

Since  there does not appear to be any significant difference between the means for manufacturer 1 and manufacturer 3.

16. 

23 – 28  3.54

–5  3.54 = –8.54 to –1.46

17. a.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Marketing Managers | Marketing Research | Advertising |
| Sample Mean | 5 | 4.5 | 6 |
| Sample Variance | .8 | .3 | .4 |

= (5 + 4.5 + 6)/3 = 5.17

= 6(5 – 5.17)2 + 6(4.5 – 5.17) 2 + 6(6 – 5.17) 2 = 7.00

MSTR = SSTR /(*k* – 1) = 7.00/2 = 3.5

 = 5(.8) + 5(.3) + 5(.4) = 7.50

MSE = SSE /(*n*T – *k*) = 7.50/(18 – 3) = .5

*F* = MSTR /MSE = 3.5/.50 = 7.00

Using *F* table (2 degrees of freedom numerator and 15 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 7.00 is .0071

Because *p*-value  = .05, we reject the null hypothesis that the mean perception score is the same for the three groups of specialists.

b. Since there are only 3 possible pairwise comparisons we will use the Bonferroni adjustment.

** = .05/3 = .017

*t*.017/2 = *t*.0085 which is approximately *t*.01 = 2.602



1.06; no significant difference

1.06; no significant difference

1.06; significant difference

18. a.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Machine 1 | Machine 2 | Machine 3 | Machine 4 |
| Sample Mean | 7.1 | 9.1 | 9.9 | 11.4 |
| Sample Variance | 1.21 | .93 | .70 | 1.02 |

= (7.1 + 9.1 + 9.9 + 11.4)/4 = 9.38

= 6(7.1 – 9.38) 2 + 6(9.1 – 9.38) 2 + 6(9.9 – 9.38) 2 + 6(11.4 – 9.38) 2 = 57.77

MSTR = SSTR /(*k* – 1) = 57.77/3 = 19.26

 = 5(1.21) + 5(.93) + 5(.70) + 5(1.02) = 19.30

MSE = SSE /(*n*T – *k*) = 19.30/(24 – 4) = .97

*F* = MSTR /MSE = 19.26/.97 = 19.86

Using *F* table (3 degrees of freedom numerator and 20 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 19.86 is .0000.

Because *p*-value  = .05, we reject the null hypothesis that the mean time between breakdowns is the same for the four machines.

b. Note: *t*α/2 is based upon 20 degrees of freedom



LSD; significant difference

19. C = 6 [(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)]

** = .05/6 = .008 and **/2 = .004

Since the smallest value for ** /2 in the *t* table is .005, we will use *t*.005 = 2.845 as an approximation for *t*.004 (20 degrees of freedom)



Thus, if the absolute value of the difference between any two sample means exceeds 1.62, there is sufficient evidence to reject the hypothesis that the corresponding population means are equal.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Means | (1,2) | (1,3) | (1,4) | (2,3) | (2,4) | (3,4) |
| | Difference | | 2 | 2.8 | 4.3 | 0.8 | 2.3 | 1.5 |
| Significant ? | Yes | Yes | Yes | No | Yes | No |

20. a. The Minitab output is shown below:

**One-way ANOVA: Attendance versus Division**

Source DF SS MS F P

Division 2 18109727 9054863 6.96 0.011

Error 11 14315319 1301393

Total 13 32425045

S = 1141 R-Sq = 55.85% R-Sq(adj) = 47.82%

Individual 95% CIs For Mean Based on Pooled StDev

Level N Mean StDev -+---------+---------+---------+--------

North 6 7702 1301 (-----\*------)

South 4 5566 1275 (-------\*-------)

West 4 8430 570 (-------\*--------)

-+---------+---------+---------+--------

4500 6000 7500 9000

Pooled StDev = 1141

Because *p*-value = .011 = .05, we reject the null hypothesis that the mean attendance values are equal.

b. *n*1 = 6 *n*2 = 4 *n*3 = 4

*t*α/2 is based upon 11 degrees of freedom

Comparing North and South



= 2136 > LSD; significant difference

Comparing North and West



= 728 < LSD; no significant difference

Comparing South and West



= 2864 > LSD; significant difference

The difference in the mean attendance among the three divisions is due to the low attendance in the South division.

21. Treatment Means:

= 13.6 = 11.0 = 10.6

Block Means:

= 9 = 7.67 = 15.67 = 18.67 = 7.67

Overall Mean:

= 176/15 = 11.73

Step 1

= (10 – 11.73) 2 + (9 – 11.73) 2 + · · · + (8 – 11.73) 2 = 354.93

Step 2

= 5 [ (13.6 – 11.73) 2 + (11.0 – 11.73) 2 + (10.6 – 11.73) 2 ] = 26.53

Step 3

= 3 [ (9 – 11.73) 2 + (7.67 – 11.73) 2 + (15.67 – 11.73) 2 +

(18.67 – 11.73) 2 + (7.67 – 11.73) 2 ] = 312.32

Step 4

SSE = SST – SSTR – SSBL = 354.93 – 26.53 – 312.32 = 16.08

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 26.53 | 2 | 13.27 | 6.60 | .0203 |
| Blocks | 312.32 | 4 | 78.08 |  |  |
| Error | 16.08 | 8 | 2.01 |  |  |
| Total | 354.93 | 14 |  |  |  |

Using *F* table (2 degrees of freedom numerator and 8 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 6.60 is .0203.

Because *p*-value = .05, we reject the null hypothesis that the means of the three treatments are equal.

22.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 310 | 4 | 77.5 | 17.69 | .0005 |
| Blocks | 85 | 2 | 42.5 |  |  |
| Error | 35 | 8 | 4.38 |  |  |
| Total | 430 | 14 |  |  |  |

Using *F* table (4 degrees of freedom numerator and 8 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 17.69 is .0005.

Because *p*-value  = .05, we reject the null hypothesis that the means of the treatments are equal.

23.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 900 | 3 | 300 | 12.60 | .0001 |
| Blocks | 400 | 7 | 57.14 |  |  |
| Error | 500 | 21 | 23.81 |  |  |
| Total | 1800 | 31 |  |  |  |

Using *F* table (3 degrees of freedom numerator and 21 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 12.60 is .0001.

Because *p*-value = .05, we reject the null hypothesis that the means of the treatments are equal.

24. Treatment Means:

= 56 = 44

Block Means:

= 46 = 49.5 = 54.5

Overall Mean:

= 300/6 = 50

Step 1

 = (50 – 50) 2 + (42 – 50) 2 + · · · + (46 – 50) 2 = 310

Step 2

= 3 [ (56 – 50) 2 + (44 – 50) 2 ] = 216

Step 3

= 2 [ (46 – 50) 2 + (49.5 – 50) 2 + (54.5 – 50) 2 ] = 73

Step 4

SSE = SST – SSTR – SSBL = 310 – 216 – 73 = 21

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 216 | 1 | 216 | 20.57 | .0453 |
| Blocks | 73 | 2 | 36.5 |  |  |
| Error | 21 | 2 | 10.5 |  |  |
| Total | 310 | 5 |  |  |  |

Using *F* table (1 degree of freedom numerator and 2 denominator), *p*-value is between .025 and .05

Using Excel or Minitab, the *p*-value corresponding to *F* = 20.57 is .0453.

Because *p*-value = .05, we reject the null hypothesis that the mean tune-up times are the same for both analyzers.

25. The blocks correspond to the 11 metropolitan areas and the treatments correspond to the 3 brands of gasoline.

The Minitab two-way ANOVA output follows.

Source DF SS MS F P

Metropolitan Area 10 0.108006 0.0108006 8.30 0.000

Brand 2 0.015836 0.0079182 6.08 0.009

Error 20 0.026030 0.0013015

Total 32 0.149873

S = 0.03608 R-Sq = 82.63% R-Sq(adj) = 72.21%

Because the *p*-value for Brand (.009) is less than ** = .05, there is a significant difference in the mean price per gallon for regular gasoline among the three brands.

26. a. Treatment Means:

= 502 = 515 = 494

Block Means:

= 530 = 590 = 458 = 560 = 448 = 436

Overall Mean:

= 9066/18 = 503.67

Step 1

= (526 – 503.67) 2 + (534 – 503.67) 2 + · · · + (420 – 503.67) 2 = 65,798

Step 2

= 6[ (502 – 503.67) 2 + (515 – 503.67) 2 + (494 – 503.67) 2 ] = 1348

Step 3

= 3 [ (530 – 503.67) 2 + (590 – 503.67) 2 + · · · + (436 – 503.67) 2 ] = 63,250

Step 4

SSE = SST – SSTR – SSBL = 65,798– 1348– 63,250= 1200

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatments | 1348 | 2 | 674 | 5.62 | .0231 |
| Blocks | 63,250 | 5 | 12,650 |  |  |
| Error | 1200 | 10 | 120 |  |  |
| Total | 65,798 | 17 |  |  |  |

Using *F* table (2 degrees of freedom numerator and 10 denominator), *p*-value is between .01 and .025.

Using Excel or Minitab, the *p*-value corresponding to *F* = 5.62 is .0231.

Because *p*-value = .05, we reject the null hypothesis that the mean scores for the three parts of the SAT are equal.

b. The mean test scores for the three sections are 502 for critical reading; 515 for mathematics; and 494 for writing. Because the writing section has the lowest average score, this section appears to give the students the most trouble.

27. The Minitab output is shown below.

**Two-way ANOVA: Heart Rate versus Method, Subject**

Source DF SS MS F P

Method 3 19805.2 6601.73 22.42 0.000

Subject 9 2795.6 310.62 1.06 0.425

Error 27 7948.8 294.40

Total 39 30549.6

S = 17.16 R-Sq = 73.98% R-Sq(adj) = 62.42%

The *p*-value corresponding to Method is .000; because the *p*-value = .000 < .05, there is a significant difference in the mean heart rate among the four methods tested.

28.



Step 1

= (135 – 111) 2 + (165 – 111) 2 + · · · + (136 – 111) 2 = 9,028

Step 2

= 3 (2) [ (104 – 111) 2 + (118 – 111) 2 ] = 588

Step 3

= 2 (2) [ (130 – 111) 2 + (97 – 111) 2 + (106 – 111) 2 ] = 2,328

Step 4

= 2 [ (150 – 104 – 130 + 111) 2 + (78 – 104 – 97 + 111) 2 +

· · · + (128 – 118 – 106 + 111) 2 ] = 4,392

Step 5

SSE = SST – SSA – SSB – SSAB = 9,028 – 588 – 2,328 – 4,392 = 1,720

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 588 | 1 | 588 | 2.05 | .2022 |
| Factor B | 2328 | 2 | 1164 | 4.06 | .0767 |
| Interaction | 4392 | 2 | 2196 | 7.66 | .0223 |
| Error | 1720 | 6 | 286.67 |  |  |
| Total | 9028 | 11 |  |  |  |

Factor A: *F* = 2.05

Using *F* table (1 degree of freedom numerator and 6 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = 2.05 is .2022.

Because *p*-value > = .05, Factor A is not significant

Factor B: *F* = 4.06

Using *F* table (2 degrees of freedom numerator and 6 denominator), *p*-value is between .05 and .10

Using Excel or Minitab, the *p*-value corresponding to *F* = 4.06 is .0767.

Because *p*-value >= .05, Factor B is not significant

Interaction: *F* = 7.66

Using *F* table (2 degrees of freedom numerator and 6 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 7.66 is .0223.

Because *p*-value  = .05, Interaction is significant

29.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 26 | 3 | 8.67 | 3.72 | .0250 |
| Factor B | 23 | 2 | 11.50 | 4.94 | .0160 |
| Interaction | 175 | 6 | 29.17 | 12.52 | .0000 |
| Error | 56 | 24 | 2.33 |  |  |
| Total | 280 | 35 |  |  |  |

Using *F* table for Factor A (3 degrees of freedom numerator and 24 denominator), *p*-value is .025

Because *p*-value = .05, Factor A is significant.

Using *F* table for Factor B (2 degrees of freedom numerator and 24 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 4.94 is .0160.

Because *p*-value = .05, Factor B is significant.

Using *F* table for Interaction (6 degrees of freedom numerator and 24 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 12.52 is .0000.

Because *p*-value = .05, Interaction is significant

30. Factor A is advertising design; Factor B is size of advertisement.



Step 1

= (8 – 16) 2 + (12 – 16) 2 + (12 – 16) 2 + · · · + (14 – 16) 2 = 544

Step 2

= 2 (2) [ (10– 16) 2 + (23 – 16) 2 + (15 – 16) 2 ] = 344

Step 3

= 3 (2) [ (14 – 16) 2 + (18 – 16) 2 ] = 48

Step 4

= 2 [ (10 – 10 – 14 + 16) 2 + · · · + (16 – 15 – 18 +16) 2 ] = 56

Step 5

SSE = SST – SSA – SSB – SSAB = 544 – 344 – 48 – 56 = 96

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 344 | 2 | 172 | 172/16 = 10.75 | .0104 |
| Factor B | 48 | 1 | 48 | 48/16 = 3.00 | .1340 |
| Interaction | 56 | 2 | 28 | 28/16 = 1.75 | .2519 |
| Error | 96 | 6 | 16 |  |  |
| Total | 544 | 11 |  |  |  |

Using *F* table for Factor A (2 degrees of freedom numerator and 6 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 10.75 is .0104.

Because *p*-value = .05, Factor A is significant; there is a difference due to the type of advertisement design.

Using *F* table for Factor B (1 degree of freedom numerator and 6 denominator), *p*-value is greater than .01

Using Excel or Minitab, the *p*-value corresponding to *F* =3.00 is .1340.

Because *p*-value > = .05, Factor B is not significant; there is not a significant difference due to size of advertisement.

Using *F* table for Interaction (2 degrees of freedom numerator and 6 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = 1.75 is .2519.

Because *p*-value > = .05, Interaction is not significant.

31. Factor A is method of loading and unloading; Factor B is type of ride.



Step 1

= (41 – 47) 2 + (43 – 47) 2 + · · · + (44 – 47) 2 = 136

Step 2

= 3 (2) [ (46 – 47) 2 + (48 – 47) 2 ] = 12

Step 3

= 2 (2) [ (46 – 47) 2 + (48 – 47) 2 + (47 – 47) 2 ] = 8

Step 4

= 2 [ (41 – 46 – 46 + 47) 2 + · · · + (44 – 48 – 47 + 47) 2 ] = 56

Step 5

SSE = SST – SSA – SSB – SSAB = 136 – 12 – 8 – 56 = 60

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 12 | 1 | 12 | 12/10 = 1.2 | .3153 |
| Factor B | 8 | 2 | 4 | 4/10 = .4 | .6870 |
| Interaction | 56 | 2 | 28 | 28/10 = 2.8 | .1384 |
| Error | 60 | 6 | 10 |  |  |
| Total | 136 | 11 |  |  |  |

Using *F* table for Factor A (1 degree of freedom numerator and 6 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = 1.2 is .3153.

Because *p*-value > = .05, Factor A is not significant

Using *F* table for Factor B (2 degrees of freedom numerator and 6 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = .4 is .6870.

Because *p*-value > = .05, Factor B is not significant

Using *F* table for Interaction (2 degrees of freedom numerator and 6 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = 2.8 is .1384.

Because *p*-value > = .05, Interaction is not significant

32. Factor A is Class of vehicle tested (small car, midsize car, small SUV, and midsize SUV) and factor B is Type (hybrid or conventional). The data in tabular format follows.

|  |  |  |
| --- | --- | --- |
|  | **Hybrid** | **Conventional** |
| **Small Car** | 37 | 28 |
|  | 44 | 32 |
| **Midsize Car** | 27 | 23 |
|  | 32 | 25 |
| **Small SUV** | 27 | 21 |
|  | 28 | 22 |
| **Midsize SUV** | 23 | 19 |
|  | 24 | 18 |

Summary statistics for the above data are shown below:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Hybrid** | **Conventional** |  |
| **Small Car** | = 40.5 | = 30.0 | = 35.25 |
| **Midsize Car** | = 29.5 | = 24.0 | = 26.75 |
| **Small SUV** | = 27.5 | = 21.5 | = 24.50 |
| **Midsize SUV** | = 23.5 | = 18.5 | = 21.00 |
|  | = 30.25 | = 23.5 | = 26.875 |

Step 1

= (37 – 26.875) 2 + (44 – 26.875) 2 + · · · + (18 – 26.875) 2 = 691.75

Step 2

= 2(2) [(35.25 – 26.875) 2 + (26.75 – 26.875) 2 + (24.5– 26.875) 2 + (21.0– 26.875) 2] = 441.25

Step 3

= 4(2) [(30.25 – 26.875) 2 + (23.5 – 26.875) 2 ] = 182.25

Step 4

= 2[(37 – 35.25– 30.25 + 26.875) 2 + (28 – 35.25– 23.5+ 26.875) 2 + · · · + (18 – 21.0 – 23.5 + 26.875) 2] = 19.25

Step 5

SSE = SST – SSA – SSB – SSAB = 691.75 – 441.25 – 182.25 – 19.25 = 49

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 441.25 | 3 | 147.083 | 24.01 | .0002 |
| Factor B | 182.25 | 1 | 182.250 | 29.76 | .0006 |
| Interaction | 19.25 | 3 | 6.417 | 1.05 | .4229 |
| Error | 49.00 | 8 | 6.125 |  |  |
| Total | 691.75 | 15 |  |  |  |

Conclusions:

Factor A: Because *p*-value = .0002 < *α* = .05, Factor A (Class) is significant

Factor B: Because *p*-value = .0006 < *α* = .05, Factor B (Type) is significant

Interaction: Because *p*-value = .4229 > *α* = .05, Interaction is not significant

The class of vehicles has a significant effect on miles per gallon with cars showing more miles per gallon than SUVs. The type of vehicle also has a significant effect with hybrids having more miles per gallon than conventional vehicles. There is no evidence of a significant interaction effect.

33. Factor A is time pressure (low and moderate); Factor B is level of knowledge (naïve, declarative and procedural).

= (1.13 + 1.56 + 2.00)/3 = 1.563

= (0.48 + 1.68 + 2.86)/3 = 1.673

= (1.13 + 0.48)/2 = 0.805

= (1.56 + 1.68)/2 = 1.620

= (2.00 + 2.86)/2 = 2.43

= (1.13 + 1.56 + 2.00 + 0.48 + 1.68 + 2.86)/6 = 1.618

Step 1

SST = 327.50 (given in problem statement)

Step 2

= 3(25)[(1.563 – 1.618)2 + (1.673 – 1.618)2] = 0.4538

Step 3

= 2(25)[(0.805 – 1.618)2 + (1.62 – 1.618) 2 + (2.43 – 1.618) 2] = 66.0159

Step 4

= 25[(1.13 – 1.563 – 0.805 + 1.618) 2 + (1.56 – 1.563 – 1.62

+ 1.618) 2 + · · · + (2.86 – 1.673 – 2.43 + 1.618) 2] = 14.2525

Step 5

SSE = SST – SSA – SSB – SSAB = 327.50 – 0.4538 – 66.0159 – 14.2525

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 0.4538 | 1 | 0.4538 | 0.2648 | .6076 |
| Factor B | 66.0159 | 2 | 33.0080 | 19.2608 | .0000 |
| Interaction | 14.2525 | 2 | 7.1263 | 4.1583 | .0176 |
| Error | 246.7778 | 144 | 1.7137 |  |  |
| Total | 327.5000 | 149 |  |  |  |

Factor A: Using Excel or Minitab, the *p*-value corresponding to *F* = .2648 is .6076. Because *p*-value > = .05, Factor A (time pressure) is not significant.

Factor B: Using Excel or Minitab, the *p*-value corresponding to *F* = 19.2608 is .0000. Because *p*-value  = .05, Factor B (level of knowledge) is significant.

Interaction: Using Excel or Minitab, the *p*-value corresponding to *F* = 4.1583 is .0176. Because *p*-value = .05, Interaction is significant.

34.

|  |  |  |  |
| --- | --- | --- | --- |
|  | *x* | *y* | *z* |
| Sample Mean | 92 | 97 | 84 |
| Sample Variance | 30 | 6 | 35.33 |

= (92 + 97 + 44) /3 = 91

= 4(92 – 91) 2 + 4(97 – 91) 2 + 4(84 – 91) 2 = 344

MSTR = SSTR /(*k* – 1) = 344 /2 = 172

 = 3(30) + 3(6) + 3(35.33) = 213.99

MSE = SSE /(*n*T – *k*) = 213.99 /(12 – 3) = 23.78

*F* = MSTR /MSE = 172 /23.78 = 7.23

Using *F* table (2 degrees of freedom numerator and 9 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 7.23 is .0134.

Because *p*-value = .05, we reject the null hypothesis that the mean absorbency ratings for the three brands are equal.

35.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Lawyer | Physical Therapist | Cabinet  Maker | Systems Analyst |
| Sample Mean | 50.0 | 63.7 | 69.1 | 61.2 |
| Sample Variance | 124.22 | 164.68 | 105.88 | 136.62 |



= 10(50.0 – 61) 2 + 10(63.7 – 61) 2 + 10(69.1 – 61) 2 + 10(61.2 – 61) 2 = 1939.4

MSTR = SSTR /(*k* – 1) = 1939.4 /3 = 646.47

 = 9(124.22) + 9(164.68) + 9(105.88) + 9(136.62) = 4,782.60

MSE = SSE /(*n*T – *k*) = 4782.6 /(40 – 4) = 132.85

*F* = MSTR /MSE = 646.47 /132.85 = 4.87

Using *F* table (3 degrees of freedom numerator and 36 denominator), *p*-value is less than .01

Using Excel or Minitab, the *p*-value corresponding to *F* = 4.87 is .0061.

Because *p*-value  = .05, we reject the null hypothesis that the mean job satisfaction rating is the same for the four professions.

36. The blocks correspond to the 10 dates on which the data were collected (Date) and the treatments correspond to the four cities (City).

The Minitab two-way ANOVA output follows.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | DF | SS | MS | F | P |
| Date | 9 | 903.02 | 100.336 | 4.55 | 0.001 |
| City | 3 | 160.08 | 53.358 | 2.42 | 0.088 |
| Error | 27 | 595.68 | 22.062 |  |  |
| Total | 39 | 1658.78 |  |  |  |

Because the *p*-value for City (.0888) is greater than ** = .05, there is no significant difference in the mean ozone level among the four cities. But, if the level of significance was ** = .10, the difference would have been significant.

37. The Minitab output is shown below:

**One-way ANOVA: Midwest, Northeast, South, West**

Source DF SS MS F P

Factor 3 376.9 125.6 7.41 0.000

Error 71 1203.3 16.9

Total 74 1580.1

S = 4.117 R-Sq = 23.85% R-Sq(adj) = 20.63%

Individual 95% CIs For Mean Based on Pooled StDev

Level N Mean StDev +---------+---------+---------+---------

Midwest 16 12.081 3.607 (-------\*--------)

Northeast 16 8.363 4.194 (-------\*--------)

South 25 12.016 4.714 (------\*------)

West 18 6.989 3.522 (-------\*-------)

+---------+---------+---------+---------

5.0 7.5 10.0 12.5

Pooled StDev = 4.117

Because the *p*-value = .000 is less than *α* = .05, we reject the null hypothesis that the mean rental vacancy rate is the same for each geographic region. The mean vacancy rates were highest (over 12%) in the Midwest and the South.

38.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Method A | Method B | Method C |
| Sample Mean | 90 | 84 | 81 |
| Sample Variance | 98.00 | 168.44 | 159.78 |

= (90 + 84 + 81) /3 = 85

= 10(90 – 85) 2 + 10(84 – 85) 2 + 10(81 – 85) 2 = 420

MSTR = SSTR /(*k* – 1) = 420 /2 = 210

 = 9(98.00) + 9(168.44) + 9(159.78) = 3,836

MSE = SSE /(*n*T – *k*) = 3,836 /(30 – 3) = 142.07

*F* = MSTR /MSE = 210 /142.07 = 1.48

Using *F* table (2 degrees of freedom numerator and 27 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = 1.48 is .2455.

Because *p*-value > = .05, we can not reject the null hypothesis that the means are equal.

39. a.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Nonbrowser | Light Browser | Heavy Browser |
| Sample Mean | 4.25 | 5.25 | 5.75 |
| Sample Variance | 1.07 | 1.07 | 1.36 |

= (4.25 + 5.25 + 5.75) /3 = 5.08

= 8(4.25 – 5.08) 2 + 8(5.25 – 5.08) 2 + 8(5.75 – 5.08) 2 = 9.33

MSTR = SSTR /(*k* – 1) = 9.33 /2 = 4.67

 = 7(1.07) + 7(1.07) + 7(1.36) = 24.5

MSE = SSE /(*n*T – *k*) = 24.5 /(24 – 3) = 1.17

*F* = MSTR /MSE = 4.67 /1.17 = 3.99

Using *F* table (2 degrees of freedom numerator and 21 denominator), *p*-value is between .025 and .05

Using Excel or Minitab, the *p*-value corresponding to *F* = 3.99 is .0340.

Because *p*-value = .05, we reject the null hypothesis that the mean comfort scores are the same for the three groups.

b. 

Since the absolute value of the difference between the sample means for nonbrowsers and light browsers is , we cannot reject the null hypothesis that the two population means are equal.

40. a. Treatment Means:

= 22.8 = 24.8 = 25.80

Block Means:

= 19.67 = 25.67 = 31 = 23.67 = 22.33

Overall Mean:

= 367 /15 = 24.47

Step 1

= (18 – 24.47) 2 + (21 – 24.47) 2 + · · · + (24 – 24.47) 2 = 253.73

Step 2

= 5 [ (22.8 – 24.47) 2 + (24.8 – 24.47) 2 + (25.8 – 24.47) 2 ] = 23.33

Step 3

= 3 [ (19.67 – 24.47) 2 + (25.67 – 24.47) 2 + · · · + (22.33 – 24.47) 2 ] = 217.02

Step 4

SSE = SST – SSTR – SSBL = 253.73 – 23.33 – 217.02 = 13.38

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Treatment | 23.33 | 2 | 11.67 | 6.99 | .0175 |
| Blocks | 217.02 | 4 | 54.26 | 32.49 |  |
| Error | 13.38 | 8 | 1.67 |  |  |
| Total | 253.73 | 14 |  |  |  |

Using *F* table (2 degrees of freedom numerator and 8 denominator), *p*-value is between .01 and .025

Using Excel or Minitab, the *p*-value corresponding to *F* = 6.99 is .0175.

Because *p*-value = .05, we reject the null hypothesis that the mean miles per gallon ratings for the three brands of gasoline are equal.

b.

|  |  |  |  |
| --- | --- | --- | --- |
|  | I | II | III |
| Sample Mean | 22.8 | 24.8 | 25.8 |
| Sample Variance | 21.2 | 9.2 | 27.2 |

= (22.8 + 24.8 + 25.8) /3 = 24.47

= 5(22.8 – 24.47) 2 + 5(24.8 – 24.47) 2 + 5(25.8 – 24.47) 2 = 23.33

MSTR = SSTR /(*k* – 1) = 23.33 /2 = 11.67

 = 4(21.2) + 4(9.2) + 4(27.2) = 230.4

MSE = SSE /(*n*T – *k*) = 230.4 /(15 – 3) = 19.2

*F* = MSTR /MSE = 11.67 /19.2 = .61

Using *F* table (2 degrees of freedom numerator and 12 denominator), *p*-value is greater than .10

Using Excel or Minitab, the *p*-value corresponding to *F* = .61 is .4406.

Because *p*-value > = .05, we cannot reject the null hypothesis that the mean miles per gallon ratings for the three brands of gasoline are equal.

Thus, we must remove the block effect in order to detect a significant difference due to the brand of gasoline. The following table illustrates the relationship between the randomized block design and the completely randomized design.

|  |  |  |
| --- | --- | --- |
| Sum of Squares | Randomized  Block Design | Completely  Randomized Design |
| SST | 253.73 | 253.73 |
| SSTR | 23.33 | 23.33 |
| SSBL | 217.02 | does not exist |
| SSE | 13.38 | 230.4 |

Note that SSE for the completely randomized design is the sum of SSBL (217.02) and SSE (13.38) for the randomized block design. This illustrates that the effect of blocking is to remove the block effect from the error sum of squares; thus, the estimate of ** for the randomized block design is substantially smaller than it is for the completely randomized design.

41. The blocks correspond to the 6 months (Month) and the treatments correspond to the models of automobile (Auto).

The Minitab output is shown below:

Two-way ANOVA: Sales versus Auto, Month

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | | | | |
| Source | DF | SS | MS | F | P |
| Auto | 5 | 221576628 | 44315326 | 33.13 | 0.000 |
| Month | 5 | 180805546 | 36161109 | 27.04 | 0.000 |
| Error | 25 | 33438976 | 1337559 |  |  |
| Total | 35 | 435821149 |  |  |  |

Because the *p*-value for Auto is less than ** = .05, there is a significant difference in the mean sales per month for the six models of compact automobile.

42. The blocks correspond to the 7 weekend series (Opponent) and the treatments correspond to the days for the series (Day).

The Minitab two-way output follows.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | DF | SS | MS | F | P |
| Day | 2 | 101469683 | 50734841 | 2.69 | 0.109 |
| Opponent | 6 | 79329936 | 13221656 | 0.70 | 0.655 |
| Error | 12 | 226577122 | 18881427 |  |  |
| Total | 20 | 407376741 |  |  |  |

Because the *p*-value for Day (.109) is greater than ** = .05, there is no significant difference in the mean attendance per game for games played on Friday, Saturday, and Sunday. These data do not suggest a particular day on which the Astros should schedule these promotions.

43.



Step 1

= (8 – 13) 2 + (12 – 13) 2 + · · · + (22 – 13) 2 = 204

Step 2

= 3 (2) [ (12 – 13) 2 + (14 – 13) 2 ] = 12

Step 3

= 2 (2) [ (9 – 13) 2 + (13.5 – 13) 2 + (16.5 – 13) 2 ] = 114

Step 4

= 2 [(8 – 12 – 9 + 13) 2 + · · · + (22 – 14 – 16.5 +13) 2] = 26

Step 5

SSE = SST – SSA – SSB – SSAB = 204 – 12 – 114 – 26 = 52

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 12 | 1 | 12 | 1.38 | .2846 |
| Factor B | 114 | 2 | 57 | 6.57 | .0308 |
| Interaction | 26 | 2 | 12 | 1.50 | .2963 |
| Error | 52 | 6 | 8.67 |  |  |
| Total | 204 | 11 |  |  |  |

Factor A: Using Excel or Minitab, the *p*-value corresponding to *F* = 1.38 is .2846. Because *p*-value > = .05, Factor A (translator) is not significant.

Factor B: Using Excel or Minitab, the *p*-value corresponding to *F* = 6.57 .0308. Because *p*-value  = .05, Factor B (language translated) is significant.

Interaction: Using Excel or Minitab, the *p*-value corresponding to *F* = 1.50 is .2963. Because *p*-value > = .05, Interaction is not significant.

44.



Step 1

= (30 – 26.75) 2 + (34 – 26.75) 2 + · · · + (28 – 26.75) 2 = 151.5

Step 2

= 2 (2) [ (30 – 26.75) 2 + (23.5 – 26.75) 2 ] = 84.5

Step 3

= 2 (2) [ (26.5 – 26.75) 2 + (27 – 26.75) 2 ] = 0.5

Step 4

= 2[(30 – 30 – 26.5 + 26.75) 2 + · · · + (28 – 23.5 – 27 + 26.75) 2]

= 40.5

Step 5

SSE = SST – SSA – SSB – SSAB = 151.5 – 84.5 – 0.5 – 40.5 = 26

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source  of Variation | Sum  of Squares | Degrees  of Freedom | Mean  Square | *F* | *p-*value |
| Factor A | 84.5 | 1 | 84.5 | 13 | .0226 |
| Factor B | .5 | 1 | .5 | .08 | .7913 |
| Interaction | 40.5 | 1 | 40.5 | 6.23 | .0671 |
| Error | 26 | 4 | 6.5 |  |  |
| Total | 151.5 | 7 |  |  |  |

Factor A: Using Excel or Minitab, the *p*-value corresponding to *F* = 13 is .0226. Because *p*-value = .05, Factor A (machine) is significant.

Factor B: Using Excel or Minitab, the *p*-value corresponding to *F* = .08 is .7913. Because *p*-value > = .05, Factor B (loading system) is not significant.

Interaction: Using Excel or Minitab, the *p*-value corresponding to *F* = 6.23 is .0671. Because *p*-value > = .05, Interaction is not significant.